USN | | | | | | | | | | |

15MATDIP41

Fourth Semester B.E. Degree Examination, Aug./Sept. 2020
Additional Mathematics – II

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Find the rank of the matrix,

$$\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & -1 \end{bmatrix}$$

By reducing it to the echelon form. **(05 Marks)**

b. Solve the following system of equations by Gauss Elimination method.

$$4x + y + z = 4$$
$$x + 4y - 2z = 4$$
$$3x + 2y - 4z = 6$$
(05 Marks)

c. Find all the eigen values and the eigen vector corresponding to the least eigen value of the matix.

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$
(06 Marks)

OR

2 a. Find the rank of the matrix,

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$

By applying elementary row transformations. **(05 Marks)**

b. Solve the following system of equations, by Gauss-Elimination method:

$$x + 2y + z = 3,$$
$$2x + 3y + 3z = 10,$$
$$3x - y + 2z = 13$$
(05 Marks)

c. Using Cayley-Hamilton theorem, find the inverse of the matrix,

$$\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$$
(06 Marks)

Module-2

3 a. Solve : $(D^2 - 6D + 9)y = e^x + e^{3x}$ **(05 Marks)**

b. Solve : $(D^2 + 3D + 2)y = 1 + 3x + x^2$ **(05 Marks)**

c. Using the method of variation of parameters, solve : $(D^2 + 1)y = \sec x \tan x$. **(06 Marks)**

OR

4 a. Solve : $(D^3 - 5D^2 + 8D - 4)y = e^{2x}$. **(05 Marks)**

b. Solve : $(D^2 - 2D + 4)y = e^x \cos x$. **(05 Marks)**

c. By the method of undetermined coefficients, solve : $(D^2 - D - 2)y = 10\sin x$. **(06 Marks)**

Module-3

5 a. Find the Laplace transform of,
(i) $\sin^2 2t$ (ii) $e^{-t}(3\sinh 2t - 2\cosh 3t)$ **(05 Marks)**

b. Find $L\left\{\frac{\cos at - \cos bt}{t}\right\}$. **(05 Marks)**

c. If $f(t) = t^2,\ 0 < t < 2$ and $f(t+2) = f(t)$ for $t > 2$. Find $\alpha\{f(t)\}$. **(06 Marks)**

OR

6 a. Find $L\{\sin t \sin 2t \sin 3t\}$. **(05 Marks)**

b. Find (i) $L\{te^{-t}\sin 4t\}$ (ii) $L\left\{\int_0^t e^{-t}\cos t\,dt\right\}$. **(05 Marks)**

c. Express $f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \sin t, & t > \pi \end{cases}$ in terms of unit-step function and hence find $L\{f(t)\}$. **(06 Marks)**

Module-4

7 a. Find the inverse Laplace transform of :
(i) $\frac{3s-4}{16-s^2}$ (ii) $\frac{s}{s^2-a^2}$ **(06 Marks)**

b. Find $L^{-1}\left\{\frac{3s+7}{s^2-2s-3}\right\}$ **(05 Marks)**

c. Solve the equation, $y'' + 4y' + 3y = e^{-t}$, with $y(0) = 1,\ y'(0) = 1$, using Laplace transforms. **(05 Marks)**

OR

8 a. Find $L^{-1}\left\{\frac{5s+3}{(s-1)(s^2+2s+5)}\right\}$. **(06 Marks)**

b. Find $L^{-1}\left\{\log\left(\frac{s^2+a^2}{s^2+b^2}\right)\right\}$. **(05 Marks)**

c. Solve the equation $y'' + 6y' + 9y = 12t^2e^{-3t}$, with $y(0) = y'(0) = 0$, using Laplace transforms. **(05 Marks)**

Module-5

9 a. For any two events A and B, prove that
(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
(ii) $P(\overline{A} \cap B) = P(B) - P(A \cap B)$ **(05 Marks)**

b. Given P(A) = 0.4, $P\left(\frac{B}{A}\right) = 0.9$ and $P\left(\frac{\overline{B}}{\overline{A}}\right) = 0.6$, find $P\left(\frac{A}{B}\right)$ and $P\left(\frac{A}{\overline{B}}\right)$. **(06 Marks)**

c. State and prove Bayes's theorem. **(05 Marks)**

OR

10 a. Let A and B be events with $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{4}$, $P(\overline{B}) = \frac{5}{8}$. Find $P(A \cap B)$, $P(\overline{A} \cap \overline{B})$, $P(\overline{A} \cup \overline{B})$ and $P(B \cap \overline{A})$. **(06 Marks)**

b. In a certain engineering college, 25% of First semester students have failed in Mathematics, 15% have failed in Chemistry and 10% have failed in both Mathematics and Chemistry. A student is selected at random.

(i) If he has failed in Chemistry, what is the probability that he has failed in Mathematics?

(ii) If he has failed in Mathematics, what is the probability that he has failed in Chemistry? **(05 Marks)**

c. Three machines A, B and C produce respectively 60%, 30%, 10% of total number of items in a factory. Percentage of defective output of these machines are respectively 2%, 3% and 4%. An item selected at random is found to be defective. Find the probability that it is produced by machine C. **(05 Marks)**

* * * * *